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Valuation Method of Equity-based Security Token Offerings (STO) for Start-Up Companies

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None declared.**Abstract**

In this paper, we develop a novel valuation method of equity-based securities token offerings (STO) for start-up companies. The closed-form discount rate function discovered in this paper is time-dependent and piecewise. The first part of the function is exponential; the second part is a power function. The reason is that, in the early years, the probability of survival of start-up firms descends more rapidly than in late years. The probability of survival function discovered has a remarkably good fit with empirical data- for the total of firms and ten industry sectors for which data is available. For the total of firms, we found that the highest discount rate has a 27.0 to 31.8% range when the liquidation value of the non-surviving start-up project is zero; this is considerably higher than observed discount rates of projects for mature firms (7.5%) but considerably less than some published discount rates for start-up projects financed by Venture Capital firms (40.6 to 70% range). To demonstrate the model, we work a valuation example in section six. A valuation method for equity STOs will help to develop a more transparent market for start-ups wanting to raise capital. Most importantly, our results show that for many start-up firms, equity STOs could be an economical alternative to raise capital.

Keywords: *discount rate, Valuation, startup, security token, security token offering, STO, Venture Capital***JEL Classifications:** *D02, D71, H11, P16, P48, P50***1. Introduction**

An equity token is a new security class, initially created with the purpose of providing early access to capital for start-ups and growth companies. Equity tokens are digital representations of company shares, and their holders are collectively the owners of the company. By definition, equity STOs are classified as securities in most jurisdictions; this certainty of classification is good for all stakeholders. One fundamental characteristic of equity tokens is that they live in a blockchain, and because of that, equity STOs trade in exchanges with blockchain facilities located in jurisdictions that permit their existence and trading.

For the valuation of companies, the DCF method is many times preferred to others because it enables the understanding of the dynamics of the business at a level of detail not present in other techniques. For the valuation of equity STOs of start-up firms using the DCF method, we need to build a framework that calculates the discount rate and forecasts the cash flows. Before we forecast cash flows, we need to dimension the opportunity facing the firm: first

considering the broadest market measure: the Total Addressable Market (TAM), from there we narrow it down to the Serviceable Available Market (SAM) and finally to the Serviceable Obtainable Market, the market that the start-up can realistically address. Later by taking into account variables such as the growth of SOM, the price and price growth of the provided good and the sales growth curve profile, we can develop a forecast. A good forecast doesn't pose any theoretical difficulty in its method, and it is of paramount importance for quality valuations. The main obstacle to build a framework for valuing start-ups using the DCF method is to calculate the project's discount rate. In the corpus of financial theory, there is no explicit formula, that we are aware of, to calculate the discount rate for start-ups, we will dwell into this issue in section 2.

In sections 3 and 4, we will go to great lengths to develop a discount rate formula for start-up firms. In section 5, we will explain our views regarding how cash flows should be calculated to arrive at the valuation of the firm. In section 6 we go through a worked example.

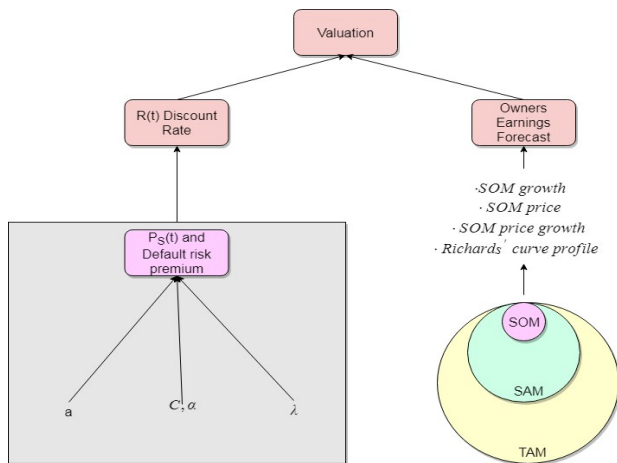


Figure 1: Map of valuation model.

2. Discount Rates Variations for Start-ups

The long-standing issue about variations in discount rates was well captured by John H. Cochrane [1] in his August 2011 Presidential Address to the American Finance Association, he stated in his conclusion:

“Discount rates vary a lot more than we thought. Most of the puzzles and anomalies that we face amount to discount-rate variations we do not understand. Our theoretical controversies are about how discount rates are formed. We need to recognize and incorporate discount-rate variation in applied procedures. We are only beginning these tasks. The facts about discount-rate variation need at least a dramatic consolidation. Theories are in their infancy.”

The fact that venture capitalists use high discount rates was addressed by Sanjai Bhagat [2] in 2014. In Bhagat’s paper summary, he explained:

“Venture capitalists typically use discount rates that are high compared to historical rates of return on common stock and other financial assets. Such high discount rates also cannot be explained in the context of any existing asset pricing theory”

In another study, Aswath Damodaran [3] mentions in a 2009 paper that Venture Capital firms have typical target rates of return of 50-70% for start-ups and suggests that these target rates must have incorporated some survival risks, Damodaran says:

“How do we know that these rates of return have survival risks built into them? In addition to the intuitive rationale that they decrease as firms move through the life cycle and the chance of failure drops, the actual returns by the venture capitalists at every stage of the process are much more modest...”

Although Damodaran’s doesn’t offer any formula to quantify his intuition, we believe that he steers in the right direction. The

probability of survival is a determinant of the formula of discount rates for start-ups as it will be shown in the next sections.

In this paper, we pose the hypothesis that we have found a novel closed-form expression of the discount rate to value start-up companies. In a quest to falsify our hypothesis, we have made extensive use of Google Scholar to search in past literature for any previous formula that calculates the discount rate for start-ups, and we have found none. This doesn’t mean that our hypothesis is correct, it only means that, until today, we have not been able to falsify our hypothesis. However, others may in the future.

3. Using historical data to determine the probability of survival function

Our objective in this section is to look at historical data and determine if we can find a probability of survival function that describes well the empirical data.

In this manuscript, we use empirical data from the Knaup and Piazza [4] (K&P) study that presented data over a 7-year period. The K&P study examined a cohort of establishments from the Bureau of Labor Statistics (BLS) Quarterly Census of Employment and Wages (QCEW) program.

We believe that the (K&P) study data is an excellent starting point to determine the probability survival function, some important characteristics of the study data are related to the comprehensiveness of the QCEW program. As presented in the K&P study, they are as follows:

- At the time the K&P study was performed, the QCEW program contained information on 8.9 million U.S. business establishments in both the public and private sector
- The monthly business establishment data is compiled on a quarterly basis for State unemployment insurance tax purposes and are edited and submitted to the BLS.
- The QCEW program collects information covering approximately 98 percent of non-farm payroll employment in the United States.
- The data generated by the QCEW program serve as the sampling frame for a range of BLS establishment surveys and as a benchmark for the Current Employment Statistics survey
- Outside researchers use QCEW microdata to investigate topics in the field of labor economics, and such data are the largest single input to the Bureau of Economic Analysis personal income accounting program. QCEW program data also are used to generate gross job flows in the Business Employment Dynamics (BED) data series.
- The QCEW program has linked data from the first quarter of 1990 through the most current quarter; the data usually are available seven months after the end of the reference quarter. The coverage and frequency of the data are unique in the

Federal statistical system in that they allow the tracking of the start-up, growth, and failure of a particular establishment concurrently with the timing of those events.

The (K&P) study follows a selected cohort of establishments from birth through 28 quarters of their lifetime, from March 1998 to March 2005, creating the basis for the 7-year survival study. The cohort data for the companies studied is in our opinion robust, and as presented in the K&P study, it has the following characteristics:

- Company births are defined as those establishments which are new in the reference quarter and show no positive employment for the previous four quarters
- Each microdata record is tested for four quarters before the reference quarter, to prevent seasonal establishments from appearing in the birth cohort.
- New establishments have no ties to any establishments that existed before the reference quarter. This approach eliminates changes in ownership from the cohort, as well as new locations of existing firms, which might be expected to behave differently from independent establishments.
- Another reason for not including new locations of existing firms is that they often represent administrative changes in the data rather than actual new locations. To include them would have risked skewing the data in terms of both survival analysis and average employment.
- The study tracked the original 212,182 new establishments across the US for the second quarter of 1998 (beginning in March of that year). The cohort accounts for approximately all births during that quarter, a typical quarter from 1992 to the end of the series.
- In the birth quarter, establishments are equivalent to firms. In subsequent quarters, establishments may be acquired by or merged with another firm, spin off a subsidiary, or open additional locations.

- Establishments that were involved in such succession relationships also were tracked across time, by following the successor establishments. Data on these successors were aggregated and assigned a unique identifier that was linked to the original birth establishment.

The resulting survival rates from the K&P study are summarized in table 1.

One salient point present in each of the industry sector series is that the survival rates descend at a decreasing rate. The descent is high in the early years and low in the later years. In this paper, we propose the hypothesis that there is an exponential function relationship in the early years and a power function relationship in the late years. The reason for this is to accommodate for the difference in descent rates between early and late years. First, we need some definitions:

- Let $P_s(t)$ be the probability of survival of the firm at time = t
- Let $P_s(0)$ be the probability of survival of the firm at time = 0
- Let t be the number of years from the date of incorporation of the firm
- Let C and α be some constants in the power function
- Let λ be some constant in the exponential function
- Let a be the transition point in time when the probability of survival function $P_s(t)$ changes from exponential to power characteristics

The proposed exponential equation for the early years has the form: $P_s(t) = P_s(0) \cdot e^{-\lambda t}$ and the proposed power function for the late years has the form: $P_s(t) = C \cdot t^{-\alpha}$. The piecewise function expressing the probability of survival looks as follows:

$$P_s(t) = \begin{cases} P_s(0) \cdot e^{-\lambda t} & \text{if } t \leq a \\ C \cdot t^{-\alpha} & \text{if } t > a \end{cases}$$

Table 1: Survival rates from the year of incorporation- US companies 1998-2005 period. (Source: Knaup & Piazza [4] and Aswath Damodaran [5])

Survival Rates from year of Incorporation								
Industry sector	Incorporation	year 1	year 2	year 3	year 4	year 5	year 6	year 7
Natural Resources and Mining	100%	82.33%	69.54%	59.41%	49.56%	43.43%	39.96%	36.68%
Construction	100%	80.69%	65.73%	53.56%	42.59%	36.96%	33.36%	29.96%
Manufacturing	100%	84.19%	68.67%	56.98%	47.41%	40.88%	37.03%	33.91%
Trade, Transportation and Utilities	100%	82.58%	66.82%	54.70%	44.68%	38.21%	34.12%	31.02%
Information	100%	80.75%	62.85%	49.49%	37.70%	31.24%	28.29%	24.78%
Financial Activities	100%	84.09%	69.57%	58.56%	49.24%	43.93%	40.34%	36.90%
Professional and Business Services	100%	82.32%	66.82%	55.13%	44.28%	38.11%	34.46%	31.08%
Education and Health Services	100%	85.59%	72.83%	63.73%	55.37%	50.09%	46.47%	43.71%
Leisure and Hospitality	100%	81.15%	64.99%	53.61%	43.76%	38.11%	34.54%	31.40%
Other Services	100%	80.72%	64.81%	53.32%	43.88%	37.05%	32.33%	28.77%
Total for all firms	100%	81.24%	65.77%	54.29%	44.36%	38.29%	34.44%	31.18%

We know that at the time of incorporation of the firm, the probability of survival is exactly 1.00, that is: $P_s(0) = 1.00$; Hence:

$$P_s(t) = \begin{cases} e^{-\lambda \cdot t} & \text{if } t \leq a \\ C \cdot t^{-\alpha} & \text{if } t > a \end{cases} \quad (1)$$

We need to find the values of parameters λ , C and α in equation (1); let us consider the first part of equation (1): $P_s(t) = e^{-\lambda \cdot t}$ if $t \leq a$. If we know one point in the function, at $t = a$, that is, point: $(a, P_s(a))$. Then it is trivial to derive λ :

$$\lambda = -\frac{\ln(P_s(a))}{a} \quad (2)$$

now, let us consider the second part of the equation (1): $P_s(t) = C \cdot t^{-\alpha}$ if $t > a$. Also, if we know two points in this function, at $t = a$ and $t = b$, that is, points: $(a, P_s(a))$ and $(b, P_s(b))$, it is trivial to derive C and α :

$$C = P_s(a) \cdot a^\alpha \quad (3)$$

and,

$$\alpha = \frac{\ln\left(\frac{P_s(a)}{P_s(b)}\right)}{\ln\left(\frac{b}{a}\right)} \quad (4)$$

With the available information in table 1, together with equations (2), (3) and (4) above and taking 3 points in each curve when $t = 0$, $t = a$, and $t = 7$ we can find the values for the parameters λ , C , and α for each industry sector and the total of all firms. The only question that remains is which value for variable a , the transition year, we should consider.

If we assume variable a is an integer; we only need to try 6 cases for a : from $a = 1$ to $a = 6$ years and observe which case offers the smoothest transition from exponential to a power function.

We did that and found that we obtain the smoothest curves when the transition point is at $t = a = 3$. The calculated results for the parameters are presented in table 2i.

For the *Total of All Firms* case, substituting parameters a , C , α and λ in equation (1), the probability of survival function looks as follows:

$$P_{s-total}(t) = \begin{cases} e^{-0.2036 \cdot t} & \text{if } t \leq 3 \\ 1.1141 \cdot t^{-0.6544} & \text{if } t > 3 \end{cases} \quad (5)$$

In Figure 2 we plot equation (5), the black dots are the results of using the empirical data from Knaup and Piazza [4] study in table 1 where time t (Years from incorporation) is in the range $0 \leq t \leq 7$ for *Total for all firms*. We get that for time $t \leq 3$ years the blue line represents the first part (exponential function) in equation (5), and for time $t > 3$ years, the red line represents the second part (power function) in equation (5). We can observe the quality of the fit and the appropriateness of using a piecewise function with a transition point at $t=3$. Fitting the empirical data with an exponential or a power function alone would not have been as good.

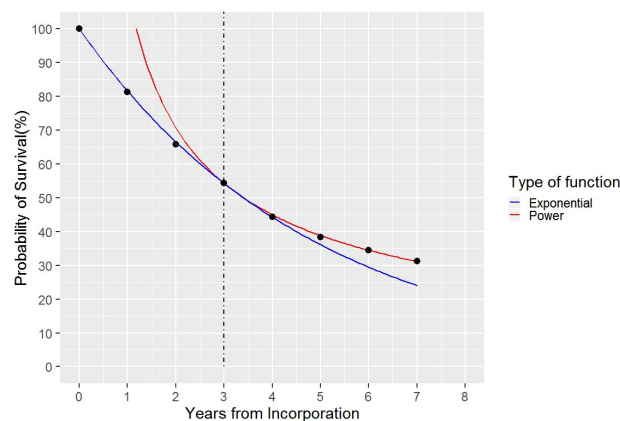


Figure 2: Probability of survival for the total for all firms- 1999-2005 period

Table 2: Parameters that define exponential and power functions- transition point at time $t = 3$

Power and Exponential Functions Parameters			
Industry Sector	C	α	λ
Natural Resources and Mining	1.1103	0.5692	0.1736
Construction	1.1373	0.6854	0.2081
Manufacturing	1.167	0.6125	0.1875
Trade, Transportation and Utilities	1.1415	0.6696	0.2011
Information	1.2131	0.8162	0.2345
Financial Activities	1.0657	0.5450	0.1784
Professional and Business Services	1.1593	0.6765	0.1985
Education and Health Services	1.0391	0.4450	0.1502
Leisure and Hospitality	1.0725	0.6312	0.2078
Other Services	1.1868	0.7282	0.2096
Total for All Firms	1.1141	0.6544	0.2036

In Figures 4 and 5 of the Appendix, we can confirm the appropriateness of piecewise function (1) to fit the empirical results for all ten industry sectors.

In section 4, parameters a , C , α and λ will serve us to calculate the discount rate necessary for valuation.

4. Method for Calculating the Discount Rate

First, let us consider some definitions for our model:

- Let r_f be the risk-free rate. For our long-term analysis, we use the returns earned by Treasury bonds
- Let r_e be the equity risk premium. As expressed by equation (6) below
- Let D be the default risk premium. It measures the additional return demanded by investors for compensation of the higher default rates historically experienced by start-ups.

There are different, some very elaborate, methods for calculating the equity risk premium. Since they don't add to the purpose of this paper, we will use the classic Capital Asset Pricing Model (CAPM) method as described by Sharpe [6] and Lintner [7]:

$$r_e = \hat{r}_e = \hat{\beta}_a \cdot (\hat{r}_m - r_f) \tag{6}$$

Where, \hat{r}_e is the expected equity risk premium for the project, $\hat{\beta}_a$ is the expected Beta of the project, and \hat{r}_m is the expected market return.

Now, let r be the discount rate to value a mature firm's project, then:

$$r = r_f + r_e \tag{7}$$

A most relevant issue is that start-ups have a much higher probability of default than mature firms. Hence, let R be the discount rate to value the start-up project that incorporates default risk premium D ; the formula looks as follows:

$$R = r_f + r_e + D \tag{8}$$

and from equations (7) and (8) we get the following expression:

$$R = r + D$$

For valuation purposes, we will incorporate the default risk via two independent methods. Method 1 incorporates the default risk as an additional risk premium in the discount rate, as in equation (8); method 2 incorporates the default risk as a probability of survival. Both valuations should throw the same result.

Note that for valuation we cannot use a method that combines methods 1 and 2, that is: a method that uses R for the discount rate and incorporates the probability of survival in the calculation as this would be double counting the default risk.

Before we start describing methods 1 and 2, let us first define some variables:

- Let V be the valuation of the firm
- Let t be the time from the date of incorporation of the firm to its exit
- Let EV be the expected Exit Value at time= t . It can be a multiple used by industry based on expected earnings or sales or a terminal value based on future earnings. The exact definition is not important as this variable will disappear in the derivation
- Let $P_s(t)$ be the probability of survival of the firm at time = t
- Let LV be the liquidation value of the firm if the firm doesn't survive
- Let r be the discount rate as calculated in equation (7). It doesn't include default risk premium D
- Let R be the discount rate as calculated in equation (8). It includes the default risk premium D

Method 1: One way of valuing a start-up is to forecast its sales or earnings sometime in the future, and then, by using a sales or earnings multiple for the industry sector, calculate an exit value (EV). Later, by discounting EV using R , one would obtain the start-up's valuation. This is a common method used in the Venture Capital industry. The formula would be as follows:

$$V = \frac{EV}{(1 + R)^t} \tag{9}$$

Method 2: Another way of valuing a start-up would be by applying the probability of survival to the exit value EV and, then, by using r as the discount rate, one gets the start-up's valuation. The complete word equation that considers a liquidation value if the firm doesn't survive is as follows:

Valuation = Probability of survival x Discounted Exit Value using discount rate r + (1-Probability of survival) x Liquidation Value of the firm which expressed in terms of the above variables looks as follows:

$$V = P_s(t) \cdot \frac{EV}{(1 + r)^t} + (1 - P_s(t)) \cdot LV \tag{10}$$

now, let F be a fraction of the liquidation value in terms of valuation V , that is: $F = \frac{LV}{V}$, then:

$$LV = F \cdot V$$

Substituting for LV in equation (10) we get:

$$V = P_s(t) \cdot \frac{EV}{(1+r)^t} + (1 - P_s(t)) \cdot F \cdot V$$

rearranging we get:

$$V - (1 - P_s(t)) \cdot F \cdot V = P_s(t) \cdot \frac{EV}{(1+r)^t}$$

$$V \cdot (1 - (1 - P_s(t)) \cdot F) = P_s(t) \cdot \frac{EV}{(1+r)^t}$$

and, thus,

$$V = \frac{P_s(t) \cdot \frac{EV}{(1+r)^t}}{1 - (1 - P_s(t)) \cdot F}$$

rearranging again, we get:

$$V = \frac{P_s(t) \cdot EV}{(1+r)^t \cdot (1 - F + F \cdot P_s(t))} \quad (12)$$

Since valuation V in equations (9) and (12) is the same, by equaling both equations we obtain the following expression:

$$\frac{EV}{(1+R)^t} = \frac{P_s(t) \cdot EV}{(1+r)^t \cdot (1 - F + F \cdot P_s(t))}$$

variable EV disappears, then, rearranging we get:

$$\frac{(1+r)^t \cdot (1 - F + F \cdot P_s(t))}{P_s(t)} = (1+R)^t$$

taking the t (th) root to both sides, we get:

$$1 + R = \sqrt[t]{\frac{(1+r)^t \cdot (1 - F + F \cdot P_s(t))}{P_s(t)}}$$

and, by further rearranging we get:

$$R = -1 + (1+r) \cdot \sqrt[t]{\frac{1-F}{P_s(t)} + F} \quad (13)$$

But we know from equation (1) that $P_s(t)$ is a piecewise function. This makes, discount rate R, a piecewise function too.

Let us consider first the part when $t \leq 3$. Substituting $P_s(t)$ by $e^{-\lambda \cdot t}$ in equation (13), we get the following expression:

$$R = -1 + (1+r) \cdot \sqrt[t]{\frac{1-F}{e^{-\lambda \cdot t}} + F}$$

alternatively,

$$R = -1 + (1+r) \cdot \sqrt[t]{e^{\lambda \cdot t} \cdot (1-F) + F}$$

Now let us consider the second part when $t > 3$. Substituting $P_s(t)$ by $C \cdot t^{-\alpha}$ in equation (13), we get the following expression:

$$R = -1 + (1+r) \cdot \sqrt[t]{\frac{1-F}{C \cdot t^{-\alpha}} + F}$$

alternatively,

$$R = -1 + (1+r) \cdot \sqrt[t]{\frac{t^\alpha}{C} \cdot (1-F) + F}$$

The complete, piecewise function, for time-dependent discount rate R(t), is as follows:

$$R(t) = \begin{cases} -1 + (1+r) \cdot \sqrt[t]{e^{\lambda \cdot t} \cdot (1-F) + F} & \text{if } t \leq a \\ -1 + (1+r) \cdot \sqrt[t]{\frac{t^\alpha}{C} \cdot (1-F) + F} & \text{if } t > a \end{cases} \quad (14)$$

Equation (14) establishes the time dependency of the discount rate. From now onwards we will use R and R(t) indistinctly, both represent the same time dependency. From equations (7) and (8) we get the equation for the default risk premium:

$$D(t) = R(t) - r \quad (15)$$

It is interesting to observe what happens in equation (14) when $F = 1$, that is, when the firm doesn't survive, but the liquidation value is equal to valuation. In such case, $R(t) = r$, thus, $D = 0$. This makes sense since if the firm gets as much from liquidation as for valuation, the default risk premium should indeed be zero.

On the other hand, if the firm doesn't survive and the liquidation value is zero, that is, $F = 0$, then, we should get the highest value for R(t). We will consider next this case for the total of all firms.

We have established the transition point in time, from exponential to a power function, at year 3. Hence, $a = 3$. From table 2 we obtain the values for the parameters: $C = 1.1142$, $\alpha = 0.6544$ and $\lambda = 0.2036$. Additionally, we assume the following values:

- Start-up covers its initial financial needs by selling equity; hence, debt is zero. Hogan and Hutson [8] found that the use of debt was rare in their study of new-technology firms. This sounds intuitively correct as start-ups have no previous record on which to base a credit application.

- Let the risk-free rate be $r_f = 2.86\%$
- The implied equity risk premium is: $\widehat{r}_m - r_f = 4.68\%$. From Damodaran's ⁱⁱⁱweb page Sept. 1st. 2018. The beta for the total of all firms is taken as for the market, that is 1.00. Hence, $r_e = 1.00 \cdot 4.68 = 4.68\%$ from equation (6)
- From previous items, $r = r_f + r_e = 2.86 + 4.68 = 7.54\%$
- Let F (the fraction of Liquidation Value/ Valuation) be 0%, as we want to evaluate the highest R(t). Note that F is endogenous to the project and requires a careful analysis of the expected liquidation value of the assets for the case in which the firm doesn't survive

From equation (14), $R(t)_{F=0}$, the R(t) function for the total of all firms when $F = 0$ is as follows:

$$R(t)_{F=0} = \begin{cases} -1 + (1 + 0.0754) \cdot e^{0.2036t} & \text{if } t \leq 3 \\ -1 + (1 + 0.0754) \cdot \sqrt[t]{\frac{t^{0.6544}}{1.1142}} & \text{if } t > 3 \end{cases} \quad (16)$$

In Figure 3 we plot $R(t)_{F=0}$ and observe how $R(t)_{F=0}$ varies with time. The black dots are the results of using the empirical data from Knaup and Piazza [4] study using equation (13). The R(t) piecewise function uses the thick blue color exponential function line for the $t \leq 3$ leg and the thick red color power function curve for the $t > 3$ leg. Once again, the piecewise R(t) function seems to be the appropriate choice

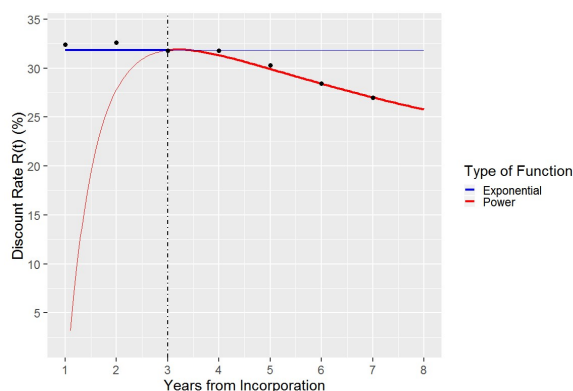


Figure 3: Plot of R(t) for Total of Firms when F=0

For years 1 to 7: $R(t)_{F=0} = \{31.8, 31.8, 31.8, 31.3, 29.9, 28.4, 27.0\}$

From Figure 3, we observe that for years 1 - 7, the range for the discount rate is 27.0 - 31.8% for the case of the total of firms when $F = 0$. This range is considerably higher than r (7.54%), the discount rate for a project in a mature firm, but much lower than the target rates applied by VC firms.

In their 1981 New England survey Wetzel [9] and Seymour reported a median compound annual rate of return demanded of 50% for start-ups by 102 individual venture investors. In 1987 Plummer [10] and Walker reported a demanded range of

discount rates of 40.6 to 59.6% for start-ups by 288 venture capital firms. In their 1991 paper, Ruhnka [11] and Young reported a mean rate of return demanded of 54.8% for start-ups by 72 venture capital firms. In his 2009 paper, Damodaran [3] mentions that typical target rates of return in VC firms for start-up projects are in the 50-70% range. From these four studies, we get from VCs a demanded rate of return in the 40.6-70% range for start-ups. We believe that the difference with our maximum range of 27.0-31.8% can be attributed to at least three factors:

- Illiquidity risk premium. Venture capital firms can only exit investments at specific moments in time: IPOs, mergers and acquisitions
- Diversification risk premium. Some VC's can only invest in one sector
- VCs provide additional services: many VCs participate in the start-ups' company boards and offer specialized services, like: coaching, advice on managerial matters, and a Rolodex full of industry contacts. These services represent costs that need to be covered for in the discount rate.

For some start-ups, the additional services provided by VC's maybe a good reason to pay for higher discount rates; others may prefer the more economic equity STO alternative. We recommend further studies on the factors that influence the difference between the ranges.

So far, we have considered that the financing for the start-up is done exclusively by selling equity and, thus, the firm has no debt. This is a reasonable assumption since Hogan and Hutson [8] found that the use of debt was rare in their study of new-technology firms. This sounds intuitively correct as start-ups have no previous record on which to base a credit application. Nevertheless, if the start-up had debt, the calculation of the discount rate to be used for valuation poses no technical difficulties; it would be equal to the weighted average of the discount rate for the un-levered firm (as calculated using equation 14) and the cost of debt. The formula would be the same as for the standard Weighted Average Cost of Capital (WACC).

5. Cash-Flow Forecast and Valuation

There is little we can add to the theory of forecasting cash-flows; it is a pretty straightforward endeavor. On the other hand, it is the task that should take most of the valuation time. It is essential that the evaluator finds, as precisely as possible, the size of the Serviceable Obtainable Market (SOM). The quality of valuation depends on finding a good measure of SOM. We don't think we can stress this enough.

We want to add that the market penetration of the products and services sold by the start-up firm will, most likely, evolve following a generalized logistic function curve (S-shaped curve), also known as Richards' [12] curve. If this is not the case, the

developer should explain why her forecast departs from this assumption. Valuation, then, would be as follows:

$$V = \sum_{i=1}^t \frac{CF \text{ to firm}_i}{(1 + R)^i} + \frac{T}{(1 + R)^t} \quad (17)$$

where the terminal value T is evaluated as follows:

$$T = \frac{CF \text{ to firm}_{t+1}}{R - g} \quad (18)$$

some definitions are as follows:

- $CF \text{ to firm}_i$ is the cash flow to the firm in year i
- t is the time horizon for which the firm is going to be evaluated
- R is the discount rate $R(t)$ as defined by equation (14) and evaluated at year t for the corresponding industry sector
- $CF \text{ to firm}_{t+1}$ is the estimated cash flow to the firm in year $t+1$
- g is the stable growth rate for $CF \text{ to Firm}$ from year $t+1$ onwards

Substituting equation (18) into (17) we get the following valuation formula:

$$V = \sum_{i=1}^t \frac{CF \text{ to firm}_i}{(1 + R)^i} + \frac{CF \text{ to firm}_{t+1}}{(1 + R)^t \cdot (R - g)} \quad (19)$$

We will use equation (19) for our worked example.

6 A worked example:

Isabel and Claire (I&C) are two young and able entrepreneurs co-founders of InsuBlock, a Blockchain life insurance company, their application is based on the Ethereum smart contracts platform. I&C have protected the intellectual property of their invention with four key patents, so they expect to start sales with a sustainable competitive advantage. I&C have a working prototype on their website, and their products are all internet based. The series of Cash Flows to firm forecast (in Millions of US dollars) for the next eight years is as follows:

$CF \text{ to Firm} = \{0.5, 5.7, 8.8, 12.9, 18.7, 26.0, 34.3, 36.2\}$ for years 1 to 8.

After year 8, the company is expected to continue with a steady CF to Firm annual growth of 3.5%. InsuBlock is considered a firm in the Financial Activities industry sector, and if the firm doesn't survive, it is believed that 10% of the initial valuation can be salvaged by selling its four patents. The company has no debt and wants to raise capital in an equity STO. The outstanding number of shares is 10 million.

Now, let's look at the value of the parameters and variables for valuation:

- We consider InsuBlock in the Financial Activities industry sector, hence, from table 2: $C = 1.0657$, $\alpha = 0.5450$, and $\lambda = 0.1784$
- $t=7$ since the 8th year cash flow to the firm is used to calculate the terminal value
- $F = 0.10$
- $g = 0.035$
- $r_f = 2.85\%$
- Implied equity risk premium, $\widehat{r}_m - r_f = 4.68\%$ and the unlevered beta, $\widehat{\beta}_a$, for the life insurance sector is 0.81, this is from Aswath Damodaran's web page Sept. 1st. 2018. Hence, $r_e = 0.81 \cdot 4.68 = 3.79\%$ from equation (6)
- $r = r_f + r_e = 2.85 + 3.79 = 6.64\%$

Since $t > 3$, we will use the second part of the $R(t)$ function in equation (14). The discount rate is as follows:

$$R(7) = -1 + (1 + 0.0664) \cdot \sqrt[7]{\frac{7^{0.5450} \cdot (1 - 0.1)}{1.0657}} + 0.1 = 21.82\%$$

We can now calculate the valuation of the company using equation (19)

Valuation =

$$\frac{0.5}{(1+0.2182)^1} + \frac{5.7}{(1+0.2182)^2} + \frac{8.8}{(1+0.2182)^3} + \frac{12.9}{(1+0.2182)^4} + \frac{18.7}{(1+0.2182)^5} + \frac{26.0}{(1+0.2182)^6} + \frac{34.3}{(1+0.2182)^7} + \frac{36.2}{(0.2182-0.035) \cdot (1+0.2182)^7}$$

= \$88.13 Million

$$\text{Price per share} = \frac{88.13}{10} = 8.81 \text{ \$/share}$$

With the above valuation, Isabel and Claire can now decide if they want to go ahead with the equity STO, and if so, which fraction of the total number of shares they want to float.

Conclusions and Recommendations

A valuation framework for equity-based STOs will allow for more transparent markets. A significant difficulty to build a DCF valuation framework is the lack of a closed-form expression of discount rates for start-up firms. In this paper, we developed a method to calculate such a discount rate; it incorporates the default risk premium present in all start-ups. The discount rate function discovered in this paper is time-dependent and piecewise. The first part of the function is

exponential; the second part is a power function. The reason is that, in the early years, the probability of survival of firms descends more rapidly than in late years. The discount rate function discovered has a remarkably good fit with empirical data- for the total of firms and for the ten industry sectors for which data is available.

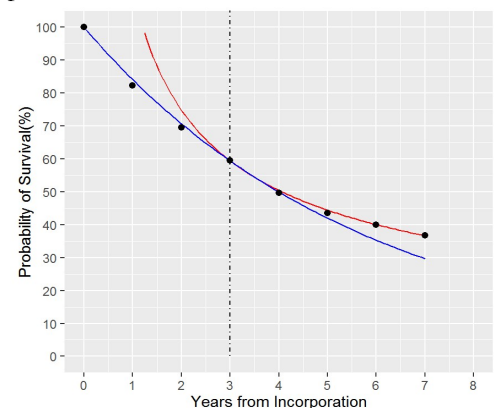
The methods to forecast the cash flows to the firm are straightforward, but the quality of the valuation will depend on the precision to measure the Serviceable Obtainable Market. Discount rates vary by industry sector. Each industry sector has its discount rate characteristics represented by parameters C , α , and λ . For future direction, we would like to suggest further work in adding data for more sectors and finer granularity of data by adding sub-sectors. Also, it would be useful to extend the model by considering variable “a” as continuous and evaluate the new optimum transition point. As the discount rate function is time-dependent, it would be useful to study its maxima-minima characteristics. A final recommendation would be of studies on the factors that influence the difference in target discount rates demanded by VC firms on start-ups and the results obtained in this manuscript.

For the total of firms, the highest discount rates were in the 27.0 to 31.8% range when the liquidation value of the non-surviving start-up project is set to zero. This range is considerably higher than observed discount rates of projects for mature firms(7.5%) but considerably less than some published discount rates for projects financed by Venture Capital firms which are in the 40.6 to 70% range. This discovery represents a positive development for the offerings of equity-based security tokens. A valuation method for equity STOs will help to develop a more transparent market for start-ups wanting to raise capital. Most importantly, our results show that for many start-up firms, equity STOs are an economical alternative to raise capital.

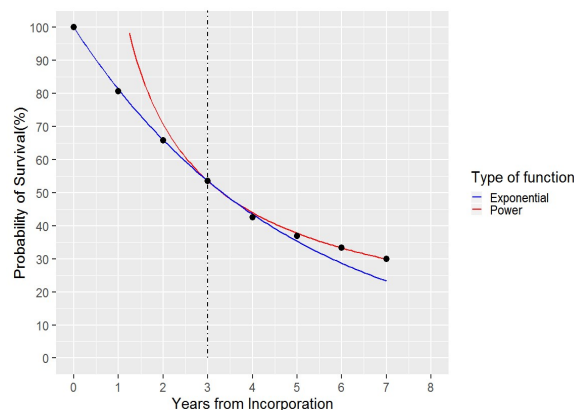
8 Appendix

Probability of survival for the ten industry sectors in the Knaup & Piazza [4] study.

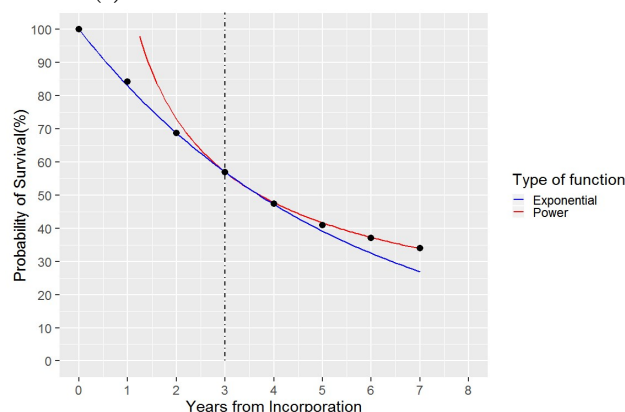
Part 1 - Probability of survival by industry sector. 1999-2005 period



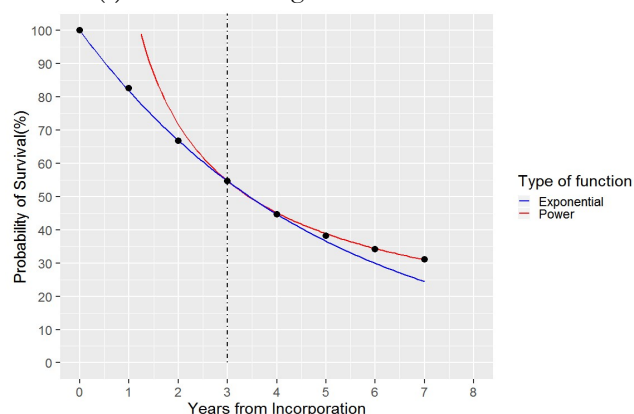
(a) Natural Resources and Mining



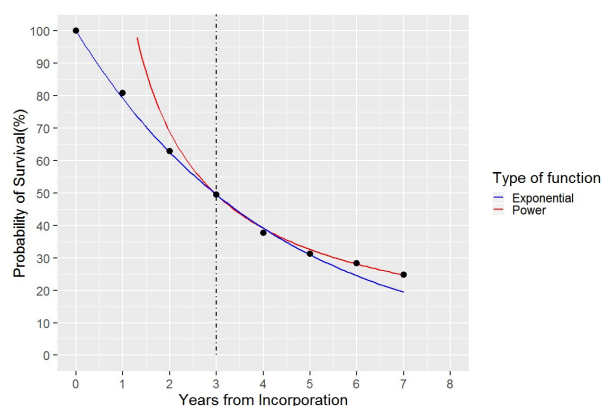
(b) Construction



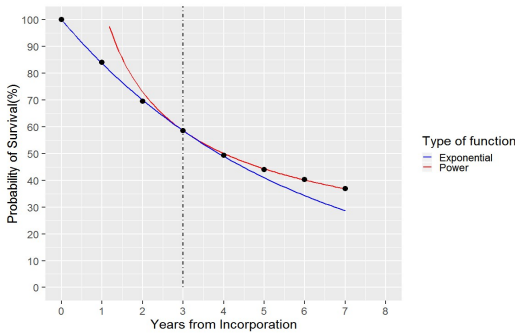
(c) Manufacturing



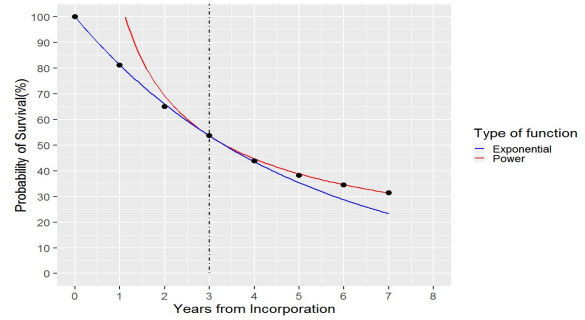
(d) Trade, Transportation and utilities



(e) Information

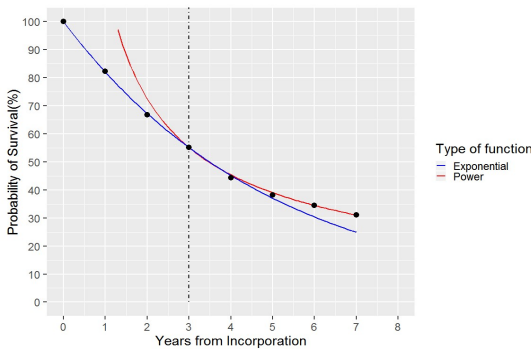


(e) Financial Activities

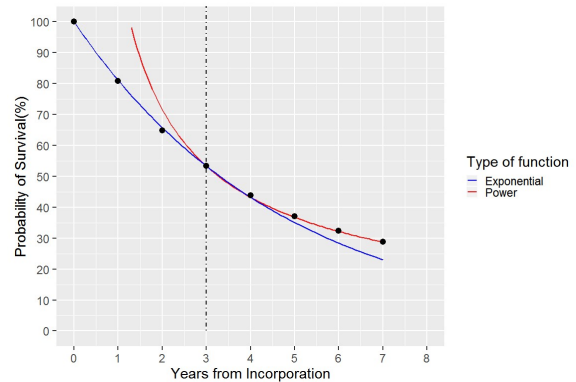


(c) Leisure and Hospitality

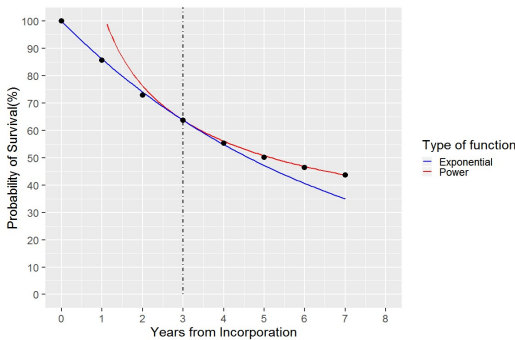
Part 2 - Probability of survival by industry sector. 1999-2005 period



(a) Professional and Business Services



(d) Other Services



(b) Education and Health Services

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ⁱ λ was calculated by using the point at $t = a = 3$ with the exponential function. And, a and C were calculated by using points at $t = a = 3$, and $t = b = 7$ with the power function

ⁱⁱ Yield 10 year Treasury on Sept. 1st. 2018

ⁱⁱⁱ <http://pages.stern.nyu.edu/~adamodar/>

^{iv} Yield 10 year Treasury on Sept. 1st. 2018

^v <http://pages.stern.nyu.edu/~adamodar/>